Data in an Uncertain World

No measurement is perfect, so we often repeat them. Each measurement or trial can be labeled with a number: 1, 2, 3 and so on. This trial number is labeled *i*, and the total number of trials is labeled N. If the quantity measured is *x*, each measurement can be labeled x_1, x_2, ... x_i, ... x_N.

In order to get our best estimate of the true value, we often ***average*** the results. To calculate the average of N measurements:

Sum the measurements and divide by how many measurements were taken

Each measurement or trial has some deviation from this average. The ***deviation*** is the difference between the average and the measurement

Absolute value of the difference between average and each individual value

Deviation = |average – measurement|

While the average is probably rather close to the proper value, it is unlikely that the average is actually equal to the proper value. There is a certain amount of ***error*** in the data. We will estimate this error with the ***average deviation***[*], which is an average of the absolute values of the deviations:

Average of the deviations

This average deviation is what we will call our ***experimental error***. When a measurement is reported, show the likely range of values by giving the average with its average deviation or experimental error. For example, a distance might be measured as "43.2 +/- 0.3 m". Usually, two measurements are considered to be in agreement (the same) if the ranges overlap. For example, "43.7 +/- 0.2 m" is in agreement with the previous example.

On a graph, the first measurement would be plotted as a point at 43.2 m with a line or ***error bar*** drawn through the point from 39.9 m to 43.5 m. A best fit line on such a graph would, ideally, pass through the error bars of most if not all the points.

When an error is given in the same units as the average, as above, it is called an ***absolute error***. Sometimes it is useful to know the size of the error relative to the average. This ***relative*** (or ***percentage***) ***error*** is calculated as a percentage of the average:

Divide Average deviation by average, multiply by 100

Relative error = (Avg Dev / Average) x 100

The Bottom Line: When practical, make multiple measurements, and report the average with either absolute or relative error.

[*] There are other measures of error, but the average deviation is appropriate for the (usually) small number of trials you will perform. It's one of the simplest measures to calculate, too!